Hedging Fixed Price Load Following Obligations in a Competitive Wholesale Electricity Market

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Typical Electricity Supply and Demand Functions

- Marginal Cost ($/MWh)
- Resource stack includes:
  - Hydro units
  - Nuclear plants
  - Coal units
  - Natural gas units
  - Misc.

- Demand (MW)
PJM Real-Time Market Prices

Daily Average RT Price ($/MWh)

Apr-98 Sep-98 Mar-99 Sep-99 Mar-00 Sep-00 Mar-01 Sep-01
Dispatch re-optimized every five minutes and locational marginal prices updated to reflect shadow prices on transmission constraints
Electricity Supply Chain

Similar exposure is faced by a trader with a fixed price load following obligation (such contracts were auctioned off in New Jersey and Montana to cover default service needs)
Both Price and Quantity are Volatile (PJM Market Price-Load Pattern)

Correlation coefficients:

0.539 for hourly price and load from 4/1998 to 3/2000 at Cal PX

0.7, 0.58, 0.53 for normalized average weekday price and load in Spain, Britain, and Scandinavia, respectively
Volumetric Risk for Load Following Obligation

- Properties of electricity demand (load)
  - Uncertain and unpredictable
  - Weather-driven → volatile
- Sources of exposure
  - Highly volatile wholesale spot price
  - Flat (regulated or contracted) retail rates & limited demand response
  - Electricity is non-storable (no inventory)
  - Electricity demand has to be served (no “busy signal”)
  - Adversely correlated wholesale price and load

Covering expected load with forward contracts will result in a contract deficit when prices are high and contract excess when prices are low resulting in a net revenue exposure due to load fluctuations
Tools for Volumetric Risk Management

- Electricity derivatives
  - Forward or futures
  - Plain-Vanilla options (puts and calls)
  - Swing options (options with flexible exercise rate)
- Temperature-based weather derivatives
  - Heating Degree Days (HDD), Cooling Degree Days (CDD)
- Power-weather Cross Commodity derivatives
  - Payouts when two conditions are met (e.g. both high temperature & high spot price)
- Demand response Programs
  - Interruptible Service Contracts
  - Real Time Pricing
Volumetric Static Hedging Model Setup

- One-period model
  - At time 0: construct a portfolio with payoff $x(p)$
  - At time 1: hedged profit $Y(p, q, x(p)) = (r-p)q + x(p)$

- Objective
  - Find a zero cost portfolio with exotic payoff which maximizes expected utility of hedged profit under no credit restrictions.

[Diagram showing the flow of market and load, with symbols for Spot market, LSE, and Portfolio for a delivery at time 1]
Mathematical Formulation

- **Objective function**

  
  \[
  \max_{x(p)} E[U[(r - p)q + x(p)]] \\
  \equiv \max_{x(p)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U[(r - p)q + x(p)] f(p, q) dq dp
  \]

  Utility function over profit

  Joint distribution of p and q

- **Constraint: zero-cost constraint**

  \[
  \frac{1}{B} E^Q[x(p)] = 0
  \]

  A contract is priced as an expected discounted payoff under risk-neutral measure

  Q: risk-neutral probability measure

  B: price of a bond paying $1 at time 1
Optimality Condition

The Lagrange multiplier is determined so that the constraint is satisfied

$$E[U'((r - p)q + x^*(p)) | p] = \lambda \cdot \frac{g(p)}{f_p(p)}$$

- Mean-variance utility function:

$$E[U(Y)] = E[Y] - \frac{1}{2} aVar(Y)$$

$$x^*(p) = \frac{1}{a} \left( 1 - \frac{g(p)}{f_p(p)} \right) + E^Q \left[ E[y(p,q) | p] \right] \frac{g(p)}{f_p(p)} - E[y(p,q) | p]$$
Illustrations of Optimal Exotic Payoffs Under Mean-Var Criterion

Bivariate lognormal distribution:

\[(\log p, \log q) \sim N(4, 0.7^2, 5.69, 0.2^2, \rho) \text{ under P & Q}\]

\[
\begin{align*}
E[p] &= $70/\text{MWh}, \sigma(p) = $56/\text{MWh} \\
E[q] &= 300\text{MWh}, \sigma(q) = 60\text{MWh}
\end{align*}
\]

\[r = $120/\text{MWh} \text{ (flat retail rate)}\]

Note: For the mean-variance utility, the optimal payoff is linear in \(p\) when correlation is 0,
Volumetric-hedging effect on profit

- Comparison of profit distribution for mean-variance utility ($\rho=0.8$)
  - Price hedge: optimal forward hedge
  - Price and quantity hedge: optimal exotic hedge

*Bivariate lognormal for ($p,q$)*

![Graph showing the distribution of profit before and after price and quantity hedging.](image)
Sensitivity to market risk premium

\[ m_2 = E^Q[\log p] \]

\[ E^Q[p] = \begin{cases} 
63.1, & \text{if } m_2 - m_1 = -0.1 \\
66.4, & \text{if } m_2 - m_1 = -0.05 \\
69.8, & \text{if } m_2 - m_1 = 0 \\
73.3, & \text{if } m_2 - m_1 = 0.05 \\
77.1, & \text{if } m_2 - m_1 = 0.1 
\end{cases} \]

*With Mean-variance utility (a = 0.0001)*
Sensitivity to risk-aversion

(Bigger ‘a’ = more risk-averse)

\[ E[U(Y)] = E[Y] - \frac{1}{2} a \text{Var}(Y) \]

*with mean-variance utility (m2 = m1 + 0.1)*

Note: if m1 = m2 (i.e., P=Q), ‘a’ doesn’t matter for the mean-variance utility.
Replication of Exotic Payoffs

\[ x(p) = x(F) \cdot 1 + x'(F)(p - F) + \int_0^F x''(K)(K - p)^+ dK + \int_F^\infty x''(K)(p - K)^+ dK \]

- **Exact replication can be obtained from**
  - a long cash position of size \( x(F) \)
  - a long forward position of size \( x'(F) \)
  - long positions of size \( x''(K) \) in puts struck at \( K \), for a continuum of \( K \) which is less than \( F \) (i.e., out-of-money puts)
  - long positions of size \( x''(K) \) in calls struck at \( K \), for a continuum of \( K \) which is larger than \( F \) (i.e., out-of-money calls)
Replicating portfolio and discretization

Payoffs from discretized portfolio
Timing of Optimal Static Hedge

**Objective function**

\[
\max_{\tau} \quad E[U((r-p_T)q_T + x_\tau(p_T))]
\]

Utility function over profit

**Subject to**

\[
E^Q[x(p_T)] = 0
\]  
zero-cost constraint

(A contract is priced as an expected discounted payoff under risk-neutral measure)

\[
dx_\tau(p_T) = \arg \max_{x(p_T)} E^Q[U((r-p_T)q_T + x(p_T))]
\]

(Same as Nasakkala and Keppo)

\[
dp_t = p_t(\mu_p(t)dt + \sigma_p(t)dB^1_t) \\
\]

\[
dq_t = q_t(\mu_q(t)dt + \sigma_p(t)dB^1_t + \sigma_q(t)dB^2_t).
\]

\[
EU((r-p_T)q_T + x_\tau(p_T)) = E[(r-p_T)q_T + x_\tau(p_T)] - \frac{1}{2}aVar((r-p_T)q_T + x_\tau(p_T))
\]
Example:

Price and quantity dynamics

\[
\frac{dp_t}{p_t} = e^{-\psi(T-t)\sigma dB_t^1} \\
\frac{dq_t}{q_t} = \phi\sigma_L dB_t^1 + \sqrt{1 - \phi^2\sigma_L dB_t^2}.
\]

The forward price and load estimate for a month one year later is assumed to be 20Euro/MWh and 1000 MWh. The following table summarizes the base values of the parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( T )</th>
<th>( r )</th>
<th>( p_0 )</th>
<th>( q_0 )</th>
<th>( \psi )</th>
<th>( \sigma )</th>
<th>( \sigma_L )</th>
<th>( \phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>1</td>
<td>40</td>
<td>20</td>
<td>1000</td>
<td>4.02</td>
<td>0.7</td>
<td>0.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>
Standard deviations vs. hedging time

![Graph showing standard deviations vs. hedging time.](image)
Standard deviation vs. hedging time

![Graph showing standard deviations of total profit for different hedging times and values of \( \phi \) and \( \psi \).]
Dependency of hedged profit distribution on timing

\[(r - p_T)q_T + x_\tau(p_T)\]

\(\tau = 0\)
\(\tau = 0.56\)
\(\tau = 0.9\)

std = 1582
std = 1802
std = 2944
Optimal hedge and replicating portfolio at optimal time

*In reality hedging portfolio will be determined at hedging time based on realized quantities and prices at that time.*
Extensions: VaR constrained hedging

- VaR is the value such that
  \[ \Pr(x \leq \text{VaR}) = \alpha \text{ for some } \alpha \text{ (typically 5%)} \]

- The VaR Constrained optimal hedging problem:
  \[ \text{Max}_{x(p)} E[Y(x)], \text{ s.t } E^Q[x(p)] = 0, \text{ VaR}(Y(x)) \geq V_0 \]

- Proposition: If the VAR is a function of the mean and variance of a distribution and is monotonically decreasing in the variance and increasing in the mean of the distribution, then a VaR constrained optimal portfolio is on the efficient mean-variance frontier.

- We prove that for the bivariate \((\ln(p), \ln(q))\) distribution the hedged profit under a Mean-Variance criteria satisfies the above proposition and hence an optimal MV hedge is equivalent to a corresponding optimal VaR constrained hedge.
Conclusion

- Risk management is an essential element of competitive electricity markets.
- The study and development of financial instruments can facilitate structuring and pricing of contracts.
- Better tools for pricing financial instruments and development of hedging strategy will increase the liquidity and efficiency of risk markets and enable replication of contracts through standardized and easily tradable instruments.
- Financial instruments can facilitate market design objectives such as mitigating risk exposure created by functional unbundling, containing market power, promoting demand response and ensuring generation adequacy.